Scientific Notation

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I. INTRODUCTION

Often in chemistry (and in other sciences) it is necessary to talk about use very small or very large numbers (for example, the distance from earth to the sun, or the speed of light). A convenient way to do this is with "scientific notation," which uses powers of ten to write very small or very large numbers. For example, the speed of light is 300,000,000 m/s, or in scientific notation:

3.00×10^8 m/s = $3.00 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$ m/s

Notice that the ten raised to the power of 8 (10^8) means to multiply the number 3.00 by 10, eight times. Notice also that each multiplication by 10 moves the decimal to the right one place, so 10^8 means to move the decimal to the right 8 times.

Scientific notation ALWAYS has EXACTLY one digit on the left of the decimal. There is no limit to the number of digits on the right of the decimal, and there may not be a digit at all. Below are several examples:

Number	Scientific Notation
10	1×10^1
22	2.2×10^1
3489	3.49×10^3
22,000	2.2×10^4
4,350.03	4.35003×10^3

The exponent on 10 may be negative or positive. A negative exponent means the number is divided by ten that number of times. For example, 0.00000200 would be:

$2.00 \times 10^{-6} = 2.00 \div 10 \div 10 \div 10 \div 10 \div 10 \div 10$

If the exponent is negative, it means the number is between 0 and 1, and the decimal must be moved to the *left* by the value of the exponent:

Number	Scientific Notation
0.1	1×10^{-1}
0.234	2.34×10^{-1}
0.003	3×10^{-3}
0.03002	3.002×10^{-2}

Practice (*answers to the following questions can be found at the end of this worksheet*)

1. Convert the following numbers from normal notation to scientific notation:
   a. 45
   b. 2,001
   c. 14.55
   d. 5,000,000
   e. 9894.00033
   f. 0.650
   g. 0.00432
   h. 0.000003
   i. 0.000022222
   j. 0.010002
2. Convert the following numbers from scientific notation to normal notation:
   a. 7×10^{5}
   b. 9.887×10^{3}
   c. 6.0034×10^{2}
   d. 2.0×10^{7}
   e. 3×10^{-1}
   f. 1.87×10^{-3}
   g. 6.0134×10^{-5}
   h. 3.0×10^{-3}

II. USING SCIENTIFIC NOTATION WITH A CALCULATOR

A required piece of equipment for this course is a calculator that can do scientific notation. There are many different models of calculators with this ability, and they may differ slightly in the way they handle scientific notation—*you should become familiar with the scientific notation features of your calculator!*

Every scientific calculator has at least the two following features that you should learn to use:

1. The ability to put the calculator in "scientific notation" mode, so that all numbers it displays are in scientific notation
2. The ability to *enter* numbers in scientific notation

Entering scientific notation mode.

First, learn how to put your calculator in scientfic notation mode. If you are unfamiliar with how to do this, consult your calculator's owner's manual. The following steps may be helpful. First, find and hit a button that says "Mode". This may require you to hit a "shift" or "2nd" key first. One of two things will likely happen:

1. You will enter a menu that allows you to choose between 2 or three modes: "Normal" (sometimes called "Float"), "Scientific", and possibly "Engineering"
   a. Select "Scientific" or "Sci"
   b. Hit "enter"
   c. Exit the menu with the "Exit" button, if hitting "enter" did not automatically exit.

2. You will see that there are 2 or 3 words you will cycle through as you hit the "Mode" key: "Norm" (or possibly "Float"), "Sci", and maybe "Eng".
   a. Continue cycling through the choices with the "Mode" key until "Sci" is highlighted or flashing
   b. Hit "enter"

If after trying these steps and consulting your owner's manual you are still unable to put your calculator in "Scientific" mode, *ask your instructor for help!*

Now that you are in scientific notation, it is important to know *how your calculator displays number in scientific notation*. Depending on your calculator, it may do this in one of several ways:

1. Using "E":
   a. The number 12, or 1.2×10^1 would be displayed as: 1.2E1
   b. The number 0.034, or 3.4×10^{-2} would be displayed as 3.4E-2
2. Using superscripts
   a. The number 12, or 1.2×10^1 would be displayed as: 1.2^{01}
   b. The number 0.034, or 3.4×10^{-2} would be displayed as 3.4^{-02}

Note: the "E" notation may be used in this course in worksheets, problem sets, etc.

Inputting numbers in Scientific Notation.

Now that you can put your calculator into scientific notation, you need to learn how to input numbers in scientific notation. The key is to find the "Exponent" key on your calculator:

1. Depending on your calculator, the "Exponent" key may be labeled as "E", "EE", "EXP", or "exp." Note: it is NOT the lowercase "e" key!
2. Depending on your calculator, you may need to use the "Shift" or "2nd" key to access the Exponent button

Note: Do **NOT** use the "^" key to enter a scientific number (e.g. 3.4 × 10^-2)!! Sometimes your calculator will do the right thing, but many times YOU WILL NOT GET THE CORRECT ANSWER!

Examples

1. After putting your calculator in Scientific Mode and returning to the entry screen, enter the following numbers / equations into your calculator and verify that you see the result shown:

Calculator Entry	Scientific Notation	Display on Calculator
100	1×10^2	1E2 or 1^{02}
101	1.01×10^2	1.01E2 or 1.01^{02}
0.1	1×10^{-1}	1E-1 or 1^{-01}
25×4	1×10^2	1E2 or 1^{02}
$25 \div 50$	5×10^{-1}	5E-1 or 5^{-01}
25 + 0.004	2.5004×10^1	2.5004E1 or 2.5004^{01}
0.004 – 25	-2.4996×10^1	-2.4996E1 or -2.4996^{01}

2. Put your calculator back into "Normal" or "Float" mode. Enter the following numbers / equations *using scientific notation* into your calculator and verify that you see the result shown:

Calculator Entry	Display on Calculator
1×10^2	100
1.01×10^2	101
1×10^{-1}	0.1
$(2.5 \times 10^2) \times 4$	100
$(5.2 \times 10^{-1}) \times 4$	2.08
$(5.2 \times 10^{-1}) \div 4$	0.13
-2.5×10^{-5}	-0.000025

Practice (*answers to the following questions can be found at the end of this worksheet*)

3. With your calculator in Scientific Mode, enter the following numbers or equations. What is the result, shown in scientific notation on your calculator?
   a. 2001
   b. 0.345
   c. 137.34
   d. - 5498.2
   e. 45×100
   f. $63 \div 900$
   g. $-8 \div 626$
   h. -43×-34
4. With your calculator in Normal (or Floating) Mode, enter the following numbers or equations *using scientific notation.* What is the result shown on your calculator?
   a. 5×10^3
   b. -9.134×10^2
   c. 1.0101×10^{-3}
   d. $(5.62 \times 10^1) \times 100$
   e. $(2.01 \times 10^{-2}) \times -4$
   f. $(4.2 \times 10^{-1}) \div 10$
   g. $(-2.5 \times 10^{-2}) \div -5$

III. ARITHMETIC WITH SCIENTIFIC NOTATION

Scientific notation involve exponents, so let's review the rules of arithmetic with exponents:

1. Multiplication and Division: when two numbers with exponents are multiplied (divided), the exponents are added (subtracted):

   $10^m \times 10^n = 10^{(m+n)}$ examples: $10^2 \times 10^4 = 10^6$ $10^2 \times 10^{-4} = 10^{-2}$

   When two numbers with exponents are divided, the exponents are subtracted:

   $10^m \div 10^n = 10^{(m-n)}$ examples: $\frac{10^5}{10^3} = 10^2$ $\frac{10^6}{10^{-3}} = 10^9$

2. Exponentials: when a number raised to an exponent is raised to another exponent, the exponents are multiplied:

   $(10^m)^n = 10^{(m \times n)}$ example: $(10^4)^2 = 10^{(4 \times 2)} = 10^8$

3. Addition and subtraction: In order to add or subtract, the exponents must be the same!

   $10^m + 10^n \neq$ cannot be done!

   We must change the numbers to align the decimals:

   example: $10^2 + 10^4 =$

   $$\begin{array}{r} 100 \\ +10000 \\ \hline 10100 \end{array}$$

With scientific notation, a number contains both the base number and the 10-exponent. Both of these must be taken into account with the arithmetic. When necessary, the final answer must be readjusted into scientific notation, with EXACTLY one digit to the left of the decimal.:

1. Multiplication and Division: when two numbers in scientific notation are multiplied, the base numbers must be multiplied AND the exponents must be added:

   $(M \times 10^m) \times (N \times 10^n) = M \times N \times 10^{(m+n)}$

   examples:

   $(2 \times 10^2) \times (3 \times 10^4) = 6 \times 10^6$ $(5 \times 10^2) \times (7.2 \times 10^{-4}) = 36 \times 10^{-2} = 3.6 \times 10^{-1}$

When two numbers in scientific notation are divided, the base numbers must be divided AND the exponents must be subtracted:

$(M \times 10^m) \div (N \times 10^n) = (M \div N) \times 10^{(m-n)}$

examples: $\frac{4\times10^6}{2\times10^3}=2\times10^3$ $\frac{2\times10^8}{4\times10^3}=5\times10^4$

2. Addition and subtraction: If the exponents are the same, the base numbers may be added or subtracted normally. Otherwise, the exponents must be made the same!

$(M \times 10^m) + (N \times 10^n) \neq$ cannot be done! $(M \times 10^m) + (N \times 10^m) = (M + N) \times (10^m)$

example: $(2 \times 10^2) + (3 \times 10^4) = \begin{array}{r} 200 \\ +30000 \\ \hline 30200 \end{array} = 3.02 \times 10^4$

Note: If you enter the numbers into your calculator using correct Scientific Notation, *your calculator will automatically perform the correct operations* on the base numbers and exponents!

Practice (*answers to the following questions can be found at the end of this worksheet*)

5. Use your calculator to do the following arithmetic operations. Give your answers in Scientific Notation (be sure to put your calculator in Scientific Mode before starting).
   a. $(4 \times 10^3) \times (3.1 \times 10^4)$
   b. $(-2.1 \times 10^2) \times (6.23 \times 10^{-4})$
   c. $(1.2 \times 10^2) \div (3.6 \times 10^4)$
   d. $(3.2 \times 10^{-4}) \div (1.6 \times 10^4)$
   e. $(6 \times 10^2) \times (6 \times 10^2)$
   f. $(6 \times 10^2)^2$
   g. $\frac{-4.5\times10^4}{5.0\times10^3}$
   h. $\frac{(1.8\times10^3)\times(2.1\times10^8)}{4.2\times10^4}$
   i. $\frac{(4\times10^6)\times(5\times10^{-2})}{(3.4\times10^2)\times(7.2\times10^{-3})}$

ANSWERS TO **PRACTICE** QUESTIONS

1. a. 4.5×10^{1} b. 2.001×10^{3} c. 1.455×10^{1} d. 5×10^{6} e. 9.89400033×10^{3}
   f. 6.50×10^{-1} g. 4.32×10^{-3} h. 3×10^{-6} i. 2.2222×10^{-5} j. 1.0002×10^{-2}
2. a. 700,000 b. 9,887 c. 600.34 d. 20,000,000 e. 0.3 f. 0.00187
   g. 0.000060134 h. 0.0030
3. a. 2.001×10^{3} b. 3.45×10^{-1} c. 1.3734×10^{2} d. 4.5×10^{3} e. 7×10^{-2}
   f. -1.2779×10^{2} g. 1.462×10^{3}
4. a. 5000 b. -913.4 c. 0.0010101 d. 5620 e. -0.0804
   f. 0.042 g. 0.0005
5. a. 1.24×10^{8} b. -1.3083×10^{-4} c. 3.3333×10^{-3} d. 2×10^{-8} e. 3.6×10^{5} f. 3.6×10^{5}
   g. -9×10^{0} h. 9×10^{6} i. 8.1699×10^{4}

MORE HELP

Overall tutorial: http://www.edinformatics.com/math_science/scinot.htm
http://www.wwnorton.com/college/chemistry/gilbert/tutorials/interface.asp?chapter=chapter_01&folder=scientific_notation
scientific notation drills: http://chemistry.csudh.edu/lechelpcs/scinotcsn7.html

